Java Exercises: Tuesday

Note: You may wish to look at the pictures on this handout on your monitor rather than in written form to see the colors.

Exercise 1: Shown below is an explanation of Proposition 1 of Book 1 of *The Elements*, which says, colloquially, that any pair of points are the vertices of an equilateral triangle.

Create a program that reads two points off the command line and creates the picture shown above. Label the initial points $A$ and $B$ and the third vertex $C$.

Exercise 2: Read three points $A$, $B$ and $C$ off the command. Draw the points, the triangle they form and its circumscribing circle. (Recall that the center of the circumscribing circle is the intersection of the perpendicular bisectors of the three sides.)

Exercise 3: The construction of the circle inscribed in a triangle is shown below.

Read the three vertices off the command line and draw this picture.
**Exercise 4:** Quadratic curves may be drawn through an elegant subdivision process. If the curve with control points $P_0$, $P_1$, and $P_2$ is parametrized by $\gamma(t)$ where $0 \leq t \leq 1$, we obtain two quadratic curves by restricting our consideration to either $0 \leq t \leq 1/2$ or $1/2 \leq t \leq 1$. The control points of these smaller curves may be easily obtained.

If we compute

$$Q_1 = (P_0 + P_1)/2$$
$$Q_2 = (P_1 + P_2)/2$$
$$Q_3 = (Q_1 + Q_2)/2,$$

then the first curve is given by the control points $P_0$, $Q_1$ and $Q_3$ while the second curve is given by $Q_3$, $Q_2$ and $P_2$. This is shown in the figure below.

(a) Create a program that takes three control points, finds the derived control points and then draws the two resulting quadratic curves.

(b) Modify the program to read an integer $N$ off the command line. Then draw an approximation to the quadratic curve by subdividing $N$ times, thus obtaining $2^N$ quadratic curves which are approximated by straight line segments. You may wish to draw also the endpoints of these segments and the original quadratic curve.

(c) Create a modification of your program in part (b) so that the approximation appears to agree with the quadratic curve to within the resolution of the monitor. This is the method by which quadratic curves are drawn.

**Exercise 5:** Draw a tiling of the plane by regular hexagons.

**Exercise 6:** (a) Draw the circle of radius 1 centered at 1 in the complex plane. Include a grid and set of axes. Also draw the image of the circle under the map $z \mapsto z^2$.

(b) Repeat the exercise above only replace the circle by a quadratic curve. One way to do this is with a FlatteningPathIterator.
**Exercise 7:** Read a real number $x$ off the command line and draw a picture to illustrate Euler’s formula

$$e^{ix} = \cos x + i\sin x.$$  

using the Taylor series:

$$e^{ix} = 1 + ix - \frac{x^2}{2} - i\frac{x^3}{3!} + \ldots$$  

Shown below is the case when $x = 1.7$. 

![Diagram](image-url)