

A Brief Introduction to Multigrid Methods

Ned W. Hummel

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Outline

1. What is Multigrid?
2. Main Ideas.
3. A Step-by-Step V-cycle.

What is Multigrid?

Multigrid methods are a class of numerical methods to (mainly) solve Elliptic PDEs.

Why are they good?

- They are fast. They perform as well as fast-elliptic solvers.
- They are general. They can be applied to nonlinear elliptic problems.

Main Ideas

There are two main parts to a multigrid method:

- Smoothing
- Coarse Grid Correction

We want to solve

$$Lu = f.$$

But we can't solve it exactly, so we get the approximate solution v .
Which gives us our error

$$e = u - v$$

If we can find e exactly, we would know the exact solution. But we can get a measure of how close v satisfies the original problem, by computing the residual

$$r = f - Lv.$$

This leads to the *residual equation*

$$Le = r$$

Now let us look at what we have:

$$v \rightarrow r = f - Lv \rightarrow Le = r \rightarrow u = v + e.$$

A Step-by-Step V-Cycle

Show Multigrid demo.

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